APPENDIX H: BASIC MATH REVIEW

If you are not using mathematics frequently it is quite normal to forget some of the basic principles. The purpose of this appendix is to provide you with a basic review of some of the mathematical principles and operations used in statistics. While this is not meant to be a comprehensive review, it should provide you with enough knowledge to feel confident in dealing with the formulas in this textbook.

1 Operations

Some of the more common mathematical operations we use in statistics include:

Operation	Meaning	Example
$+$	addition	$20 + 32 = 52$
-	subtraction	41 - 26 = 15
$\div$	division	$36 \div 2 = 18$
$\times$	multiplication	$47 \times 26 = 1{,}222$
$<$	less than	$45 < 60$
$>$	greater than	$60 > 45$
$\leq$	less than or equal to	$25 \leq 30$ or $30 \leq 30$
$\geq$	greater than or equal to	$40 \geq 35$ or $35 \geq 35$
$=$	equal to	$45 = 45$ or a = b
$\neq$	not equal to	$3 \neq 5$ or a $\neq$ b
$\approx$	approximately equal to	$3.999 \approx 4.0$
$\mid\ \mid$	absolute value of	$\lvert -6 \rvert = 6$ or $\lvert -x \rvert =$ x
∞	infinity	never ends (e.g. 1, 2, 3, ...)
$\sqrt{\ }$	square root	$\sqrt{25} = 5$
$\sum$	summation	$\sum(5, 6, -1) = 5 + 6$ - 2 = 9
$\prod$	product of	$\prod(5, 6, -2) = 5 \times 6 \times$ -2 = -60

2 Order of Operations

Suppose you have the following formula:

$$2(3+4) - 5$$

To determine the order in which you conduct the operations, you follow the BEDMAS rule. The BEDMAS rule means we do the following operations in order:

1. **B**rackets
2. **E**xponents (such as squares and square roots)
3. **D**ivision

4. **M**ultiplication
5. **A**ddition
6. **S**ubtraction

Following these rules we would get:

$$2(3+4) - 5 = 2(7) - 5 = 2 \times 7 - 5 = 14 - 5 = 9$$

A slightly more difficult example might be:

$$2(14 \div 7)^2 - \sqrt{25}$$

Using BEDMAS we would get:

$$2(2)^2 - \sqrt{25} = 2(2)^2 - 5 = 2(4) - 5 = 8 - 5 = 3$$

3 Exponents and Square Roots

3.1 Positive Exponents

An exponent is used to multiply a number (called the base) by itself. The exponent tells us how many times to multiply that number by. For example:

2^1	$=$	2	
2^2	$=$	$2 \times 2 = 4$	(squared)
2^3	$=$	$2 \times 2 \times 2 = 8$	(cubed)
2^{10}	$=$	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1{,}024$	(2 to the power of 10)

When multiplying base numbers that are the same, where each have positive exponents, such as:

$$2^2 \times 2^3$$

...you add the exponents together. For example:

$$2^2 \times 2^3 = 2(2+3) = 2^5 = 32$$

When a base number within a bracket has a positive exponent and the result is also raised to a positive exponent, such as:

$$(2^2)^3$$

...you multiply the exponents together. For example:

$$(2^2)^3 = 2^{(2 \times 3)} = 2^6 = 64$$

When dividing base numbers that are the same, where each have positive exponents, such as:

$$\frac{2^5}{2^2}$$

...you subtract the exponents. For example:

$$2^5 \div 2^2 = 2^{(5-2)} = 2^3 = 8$$

3.2 Negative Exponents

Since a positive exponent tells you how many times to multiply the base number by itself, a negative exponent tells you how many times to divide the base number by itself. For example:

$$2^{-1} = \frac{1}{2} = 0.50$$

$$2^{-2} = \frac{1}{2} \times \frac{1}{2} = 0.25$$

Likely the easiest way to deal with negative exponents is to first invert the base number, meaning make it one over the base number $\left(\frac{1}{\text{base number}}\right)$, and apply the absolute value of the exponent to the base number. For example:

$$2^{-2} = \frac{1}{2^{|-2|}} = \frac{1}{2^2} = \frac{1}{4} = 0.25$$

$$2^{-3} = \frac{1}{2^{|-3|}} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8} = 0.125$$

$$2^{-10} = \frac{1}{2^{|-10|}} = \frac{1}{2^10} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{1,024} = 0.000977$$

3.3 Square Roots

The square root of a number is the opposite (or inverse) of squaring a number. For example, the square of 5 is:

$$5^2 = 5 \times 5 = 25$$

The square root of 25 is:

$$\sqrt{25} = 5 \text{ which is the same as saying} \sqrt{5^2}$$

4 Equations with Summation ($\sum$) and Product of ($\prod$) Notation

4.1 Equations with Summation ($\sum$) Notation

The summation ($\sum$) symbol simply means to sum up the numbers. For example, suppose you had the following set of numbers which we called x:

$$x = 1, 2, 3, 4, 5$$

...and we want to write a simple way of say add all of the numbers in x together. We could write this using the sigma as follows:

$$\sum x = 1 + 2 + 3 + 4 + 5 = 15$$

If we want to add the squares of the numbers, we write

$$\sum x^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$$

If we want to add the numbers and then square the result, we write

$$\left(\sum x\right)^2 = (1 + 2 + 3 + 4 + 5)^2 = 15^2 = 225$$

If we want to add the squares of the numbers and then divide it by the sample size (n) we write

$$\frac{\sum x^2}{n} = \frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2}{5} = 11$$

4.2 Equations with Product of ($\prod$) Notation

The "product of symbol" ($\prod$) (which is the capital "pi" symbol) tells us to multiply the numbers together. For example, suppose you had the following set of numbers which we called x:

$$x = 1, 2, 3, 4, 5$$

...and we want to write a simple way of say add multiply the numbers in x together. We could write this using the sigma as follows:

$$\prod x = 1 \times 2 \times 3 \times 4 \times 5 = 120$$

If we want to multiple the squares of the numbers, we write

$$\prod x^2 = 1^2 \times 2^2 \times 3^2 \times 4^2 \times 5^2$$

If we want to multiple the squares of the numbers and then divide it by the sample size (n) we write

$$\frac{\prod x^2}{n} = \frac{1^2 \times 2^2 \times 3^2 \times 4^2 \times 5^2}{5} = \frac{14,400}{5} = 2{,}880$$

5 Working with Fractions

5.1 Adding Fractions

To add fractions together, you must first find a common denominator. For example 20 is a common denominator for ...

$$\frac{1}{4} + \frac{3}{5}$$

Once you have a common denominator, you multiply both the numerator and the denominator by the number required to get the common denominator. For example, to get the common denominator of 20, you multiply the 4 by 5 and the 5 by 4. You then do the same for their numerators. So ...

$$\frac{1}{4} + \frac{3}{5} = \frac{1 \times 5}{4 \times 5} + \frac{3 \times 4}{5 \times 4} = \frac{5}{20} + \frac{12}{20}$$

...you then add the numerators to get...

$$\frac{5}{20} + \frac{12}{20} = \frac{17}{20}$$

5.2 Subtracting Fractions

Similar to adding fractions, to subtract fractions, you must also find a common denominator. For example 12 is a common denominator for ...

$$\frac{1}{3} - \frac{1}{4}$$

Therefore...

$$\frac{1}{3} - \frac{1}{4} = \frac{1 \times 4}{3 \times 4} - \frac{1 \times 3}{4 \times 3} = \frac{4}{12} - \frac{3}{12}$$

...you then subtract the numerators to get...

$$\frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

5.3 Multiplying Fractions

To multiply fractions, you simply multiply the numerators and the denominators. For example:

$$\frac{1}{3} \times \frac{1}{4} = \frac{1 \times 1}{3 \times 4} = \frac{1}{12}$$

5.4 Dividing Fractions

To divide fractions, you take the reciprocal of the second fraction and then multiply the two. For example:

$$\frac{1}{3} \div \frac{1}{4} = \frac{1}{3} \times \frac{4}{1} = \frac{4}{3} = 1\frac{1}{3}$$

As an additional example, consider...

$$\frac{11}{32} \div \frac{3}{4} = \frac{11}{32} \times \frac{4}{3} = \frac{44}{96} \text{ or } \frac{11}{24}$$

6 Basic Algebra

A key thing to remember when working with algebra is that to solve a problem you must perform the same operation on the left side of the equation as you do on the right.

For example, suppose we had the following formula and we wanted to solve for x:

$$2x + 5 = 11$$

First we subtract 5 from both sides.

$$2x + 5 \text{ - } 5 = 11 \text{ - } 5$$

...which equals

$$2x = 6$$

To remove the 2 from the coefficient of x, we then divide both sides by 2.

$$\frac{2x}{2} = \frac{6}{2} \text{ which equals } \frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2} \text{ which equals } x = \frac{6}{2} \text{ which equals } x = 3$$

As another example, solve for x in the equation:

$$3x + 8 = 2x + 10$$

First we subtract $2x$ from both sides to get:

$$3x + 8 - 2x = 2x + 10 - 2x$$

...which equals

$$x + 8 = 10$$

Now subtract 8 from both sides.

$$x + 8 - 8 = 10 - 8$$

...which equals

$$x = 2$$

Occasionally you may want to manipulate an equation in order to solve for a specific value. For example, consider the formula for the standard error of the mean ($s_{\bar{x}}$) and you want to solve for n.

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

First you multiply both sides by $\sqrt{n}$ to get:

$$s_{\bar{x}} \times \sqrt{n} = \frac{s}{\sqrt{n}} \times \sqrt{n}$$

...which equals

$$s_{\bar{x}} \times \sqrt{n} = s$$

Next, you divide both sides by $s_{\bar{x}}$:

$$\frac{s_{\bar{x}} \times \sqrt{n}}{s_{\bar{x}}} = \frac{s}{s_{\bar{x}}}$$

...which equals

$$\sqrt{n} = \frac{s}{s_{\bar{x}}}$$

Then to remove the square roots on the left hand side, you square both sides as follows:

$$\left(\sqrt{n}\right)^2 = \left(\frac{s}{s_{\bar{x}}}\right)^2$$

...which equals

$$n = \left(\frac{s}{s_{\bar{x}}}\right)^2$$

7 Scientific Notation

Quite often you'll come across numbers written in scientific notation format. Scientific notation uses powers of 10 to express numbers, which are either very small or very large, in standard decimals. To convert a value to scientific notation, you use exponents to show how many points the decimal has been moved to the left of the right.

To go from a large number (≥ 100) to scientific notation, you move the decimal to the left. To convert back to a large number, you move the decimal to the right.

For example:

100	=	1×10^2
1,340,000	=	1.34×10^6
-60,200,000	=	-6.02×10^7
5.37×10^5	=	537,000
-6.04×10^8	=	-604,000,000

To go from a small number (≤ 0.01) to scientific notation, you move the decimal to the right. To convert back to a large number, you move the decimal to the left.

For example:

0.01	=	1×10^{-2}
0.000457	=	4.57×10^{-4}
-0.0000578	=	-5.78×10^{-5}
6.37×10^{-7}	=	0.000000637
-3.28×10^{-3}	=	-0.00328